Ranked retrieval

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Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Writing Boolean queries is hard

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1: "*standard user dlink 650*" → 200,000 hits
- Query 2: "*standard user dlink 650 no card found*": 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval, the system returns an ordering over the (top) documents in the collection for a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- Ranked list of results: No more feast or famine

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in [0, 1] – to each document
- This score measures how well document and query “match”.

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)

Jaccard coefficient

- jaccard*(A,B)* = $|A \cap B| \,/\, |A \cup B|$
- jaccard*(A,A)* = 1
- jaccard*(A,B)* = 0 if $A \cap B = 0$
- Always assigns a number between 0 and 1.

Attributes occuring in both samples	4
Attributes occuring in sample X only	1
Attributes occuring in sample Y only	2

=4/(4+1+2)

Jaccard's Similarity Index ⟶ 0.57

Issues with Jaccard for scoring

- Privileges shorter documents
  - We need a more sophisticated way of normalizing for length $|A \cap B| / \sqrt{|A \cup B|}$
- It doesn't consider *term frequency*
  - how many times a term occurs in a document
- Does not account for term *informativeness*
  - *How important is the term in the document*

Accounting for term frequency

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term frequency tf

- The term frequency $tf_{t,d}$ of term *t* in document *d* is defined as the number of times that *t* occurs in *d*.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \mathrm{tf}_{t,d}, & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- 0 → 0, 1 → 1, 2 → 1.3, 10 → 2, 1000 → 4, etc.
- Score for a document-query pair: sum over terms *t* in both *q* and *d*:
- score $= \sum_{t \in q \cap d} (1 + \log \mathrm{tf}_{t,d})$
- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high, increase, line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
  - How/when will it break?

idf weight

- df_t is the document frequency of *t*: the number of documents that contain *t*
  - df_t is an inverse measure of the informativeness of *t*
  - $df_t \leq N$
- We define the idf (inverse document frequency) of *t* by

$$idf_t = \log_{10}(N/df_t)$$

  - We use log (N/df_t) instead of N/df_t to "dampen" the effect of idf.

idf example, suppose N = 1 million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$\text{idf}_t = \log_{10}(N/\text{df}_t)$$

There is one idf value for each term t in a collection.

tf.idf weighting

- The tf.idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = \log(1 + \mathrm{tf}_{t,d}) \times \log_{10}(N / \mathrm{df}_t)$$

- Best known weighting scheme in information retrieval
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

Score for a document given a query

$$\text{Score}(q,d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

- There are many variants
  - How “tf” is computed (with/without logs)
  - Whether the terms in the query are also weighted
  - …

tf-idf weighting has many variants

Term frequency		Document frequency		Normalization	
n (natural)	$\mathrm{tf}_{t,d}$	n (no)	1	n (none)	1
l (logarithm)	$1 + \log(\mathrm{tf}_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df}_t}$	c (cosine)	$\frac{1}{\sqrt{w_1^2+w_2^2+\ldots+w_M^2}}$
a (augmented)	$0.5 + \frac{0.5 \times \mathrm{tf}_{t,d}}{\max_t(\mathrm{tf}_{t,d})}$	p (prob idf)	$\max\{0, \log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	$1/u$
b (boolean)	$\begin{cases} 1 & \text{if } \mathrm{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/CharLength^{\alpha}$, $\alpha < 1$
L (log ave)	$\frac{1+\log(\mathrm{tf}_{t,d})}{1+\log(\mathrm{ave}_{t \in d}(\mathrm{tf}_{t,d}))}$				

Columns headed ‘n’ are acronyms for weight schemes.

Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation *ddd.qqq,* using the acronyms from the previous table
  - A very standard weighting scheme is: lnc.ltc
    - Document: logarithmic tf (l as first character), no idf and cosine normalization
    - Query: logarithmic tf (l in leftmost column), idf (t in second column), no normalization …

tf-idf example: lnc.ltc

Document: *car insurance auto insurance*
Query: *best car insurance*

Term	Query						Document				Prod
	tf-raw	tf-wt	df	idf	wt	n'lize	tf-raw	tf-wt	wt	n'lize	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is *N*, the number of docs?

Doc length $= \sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$

Score = 0+0+0.27+0.53 = 0.8

Binary → count → weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

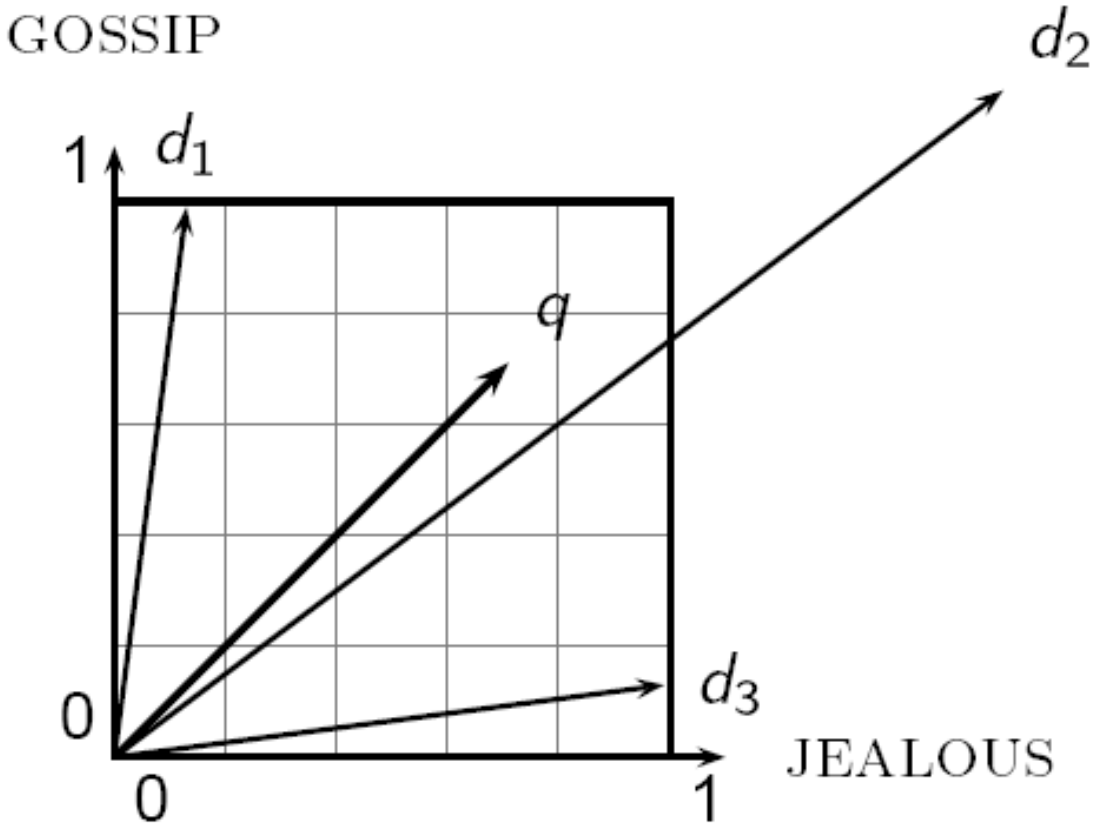
- So we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- We do this because we want to get away from the you're-either-in-or-out Boolean model.
- Instead: rank more relevant documents higher than less relevant documents

Euclidean distance is a bad idea

- The Euclidean distance between q and d_2 is large even though the distribution of terms in the query q and the distribution of terms in the document d_2 are very similar.



cosine(query,document)

Dot product

Unit vectors

$$\cos(\vec{q},\vec{d}) = \frac{\vec{q}\bullet\vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|}\bullet\frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2}\sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query
d_i is the tf-idf weight of term i in the document

$\cos(\vec{q},\vec{d})$ is the cosine similarity of $\vec{q}$ and $\vec{d}$ … or, equivalently, the cosine of the angle between $\vec{q}$ and $\vec{d}$.

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d′ (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q},\vec{d}) = \vec{q} \bullet \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

Cosine similarity amongst 3 documents

- How similar are these novels
- SaS: *Sense and Sensibility*
- PaP: *Pride and Prejudice*, and
- WH: *Wuthering Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

- Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

- After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

cos(SaS,PaP) ≈
0.789 × 0.832 + 0.515 × 0.555 + 0.335 × 0.0 + 0.0 × 0.0
≈ 0.94
cos(SaS,WH) ≈ 0.79
cos(PaP,WH) ≈ 0.69

Why do we have cos(SaS,PaP) > cos(SaS,WH)?

Computing cosine scores

```
CosineScore(q)
 1  float Scores[N] = 0
 2  float Length[N]
 3  for each query term t
 4  do calculate w_{t,q} and fetch postings list for t
 5      for each pair(d, tf_{t,d}) in postings list
 6      do Scores[d] += w_{t,d} × w_{t,q}
 7  Read the array Length
 8  for each d
 9  do Scores[d] = Scores[d]/Length[d]
10  return Top K components of Scores[]
```

Summary – vector space models

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top *K* (e.g., *K* = 10) to the user

Ranked retrieval

LANGUAGE MODELS

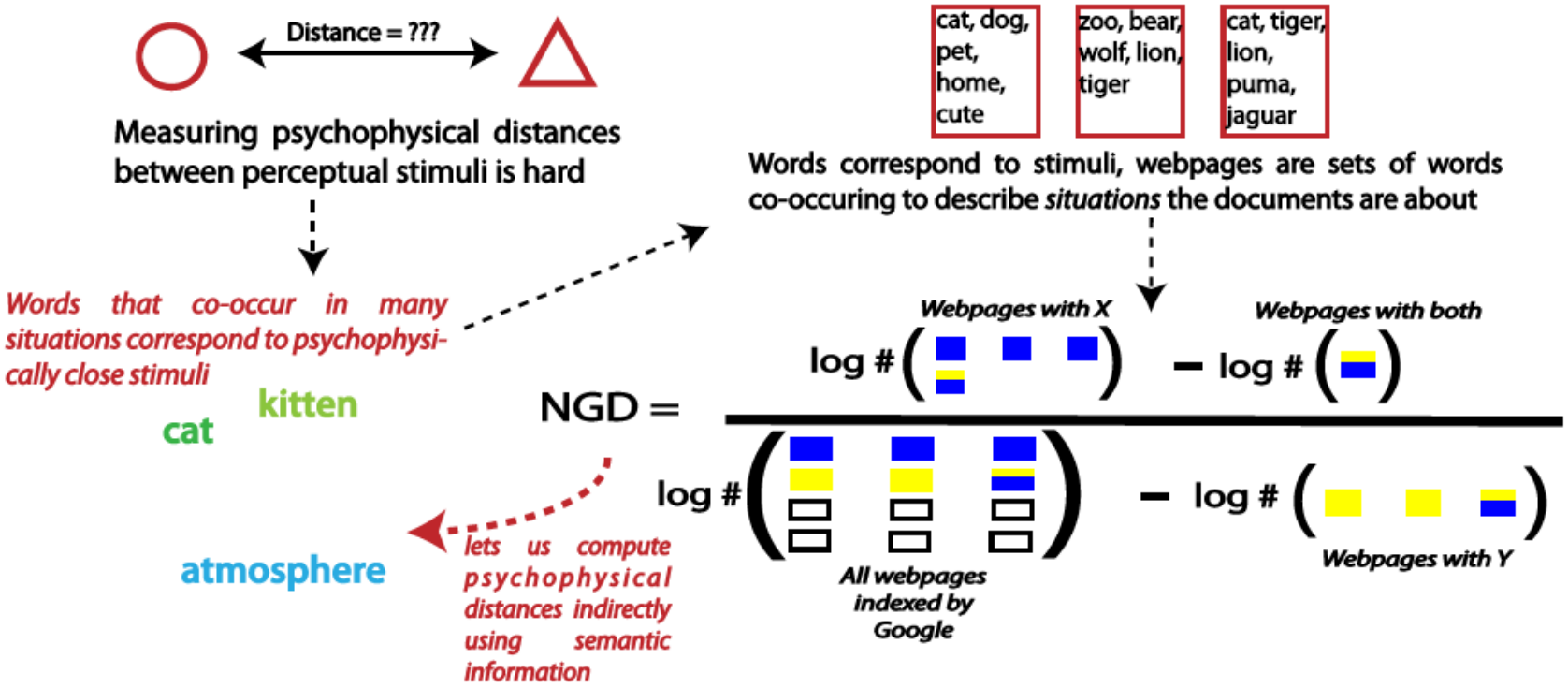
Trouble with frequency-based models

- Too literal
- Can’t deal with misspellings, synonyms etc.
- Natural language queries are hard to deal with if you don’t address these difficulties

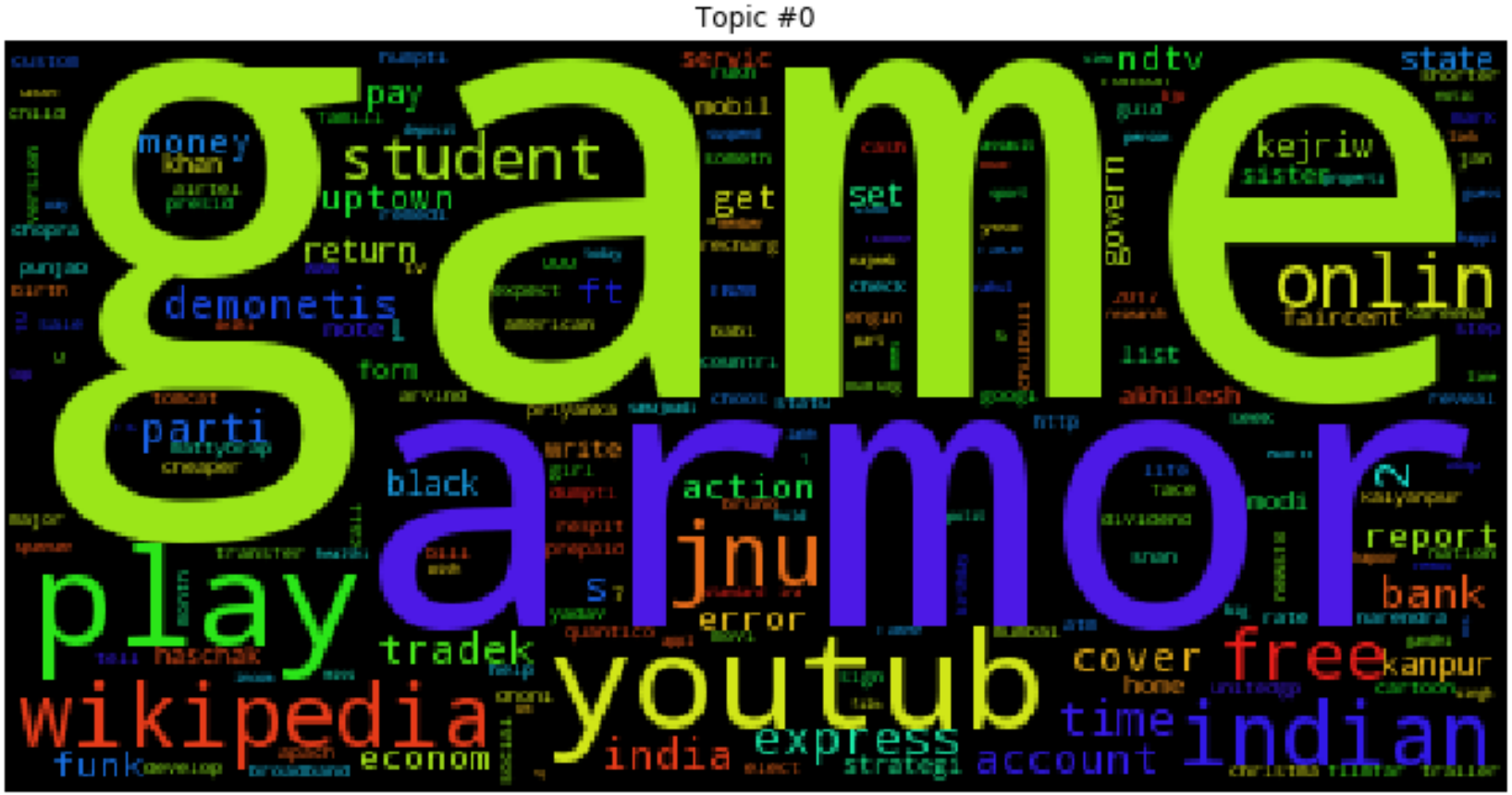
Language Model

- *Unigram language model*
  - probability distribution over the words in a language
  - generation of text consists of pulling words out of a “bucket” according to the probability distribution and replacing them
- N-gram language model
  - some applications use bigram and trigram language models where probabilities depend on previous words

Semantic distance



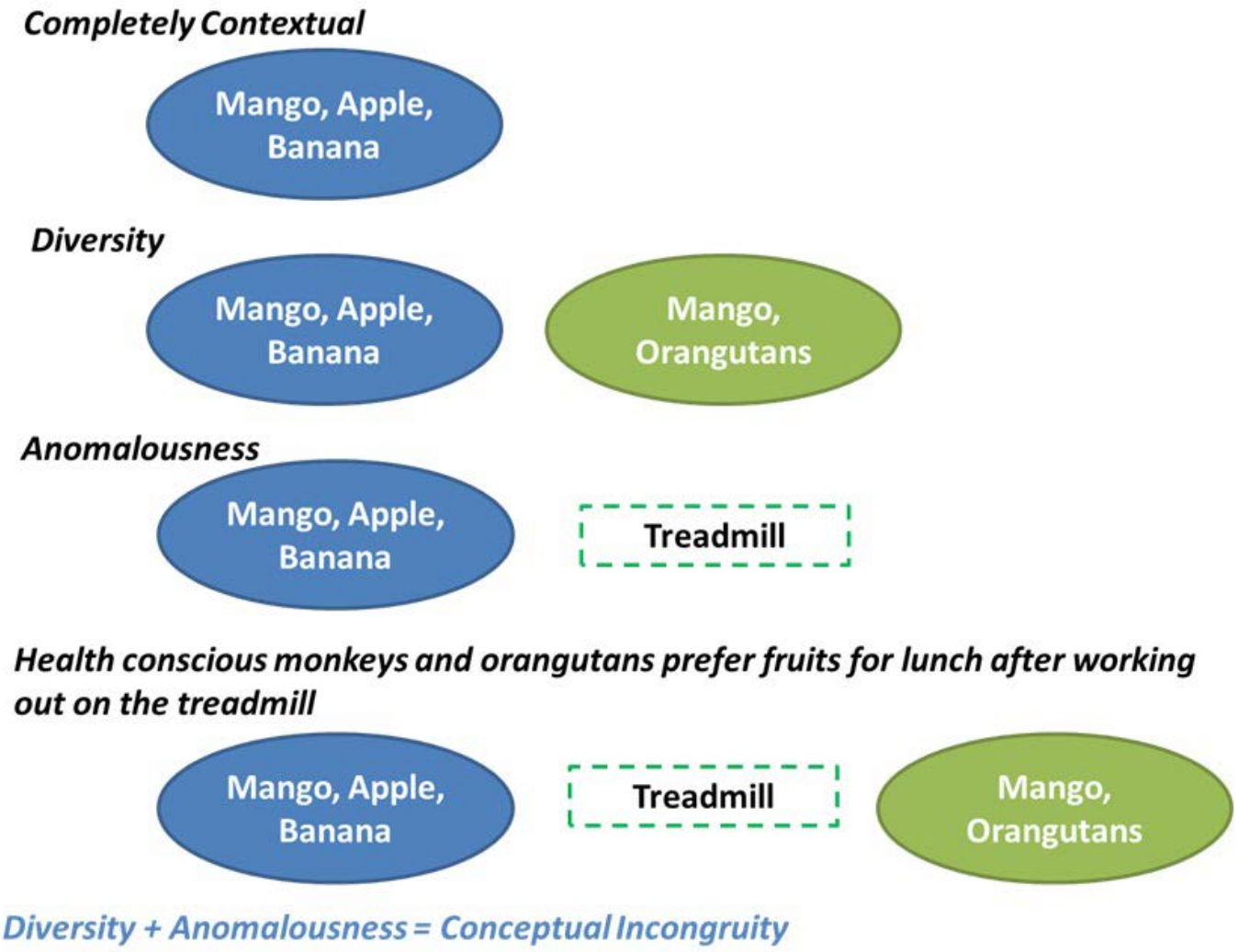
Sample topic



Language Model

- A *topic* in a document or query can be represented as a language model
  - i.e., words that tend to occur often when discussing a topic will have high probabilities in the corresponding language model
  - The basic assumption is that words cluster in semantic space
- *Multinomial* distribution over words
  - text is modeled as a finite sequence of words, where there are *t* possible words at each point in the sequence
  - commonly used, but not only possibility
  - doesn't model *burstiness*

Has interesting applications



LMs for Retrieval

- 3 possibilities:
  - probability of generating the query text from a document language model
  - probability of generating the document text from a query language model
  - comparing the language models representing the query and document topics
- Models of topical relevance

Query-Likelihood Model

- Rank documents by the probability that the query could be generated by the document model (i.e. same topic)
- Given query, start with P(D|Q)
- Using Bayes' Rule

$$p(D|Q) \overset{rank}{=} P(Q|D)P(D)$$

- Assuming prior is uniform, unigram model

$$P(Q|D) = \prod_{i=1}^{n} P(q_i|D)$$

Other query constructions

$$
\begin{aligned}
bel_{not}(q) &= 1 - p_1 \\
bel_{or}(q) &= 1 - \prod_i^n (1 - p_i) \\
bel_{and}(q) &= \prod_i^n p_i \\
bel_{wand}(q) &= \prod_i^n p_i^{wt_i} \\
bel_{max}(q) &= max\{p_1, p_2, \ldots, p_n\} \\
bel_{sum}(q) &= \frac{\sum_i^n p_i}{n} \\
bel_{wsum}(q) &= \frac{\sum_i^n wt_i p_i}{\sum_i^n wt_i}
\end{aligned}
$$

Estimating Probabilities

- Obvious estimate for unigram probabilities is

$$P(q_i|D) = \frac{f_{q_i,D}}{|D|}$$

- *Maximum likelihood estimate*
  - makes the observed value of $f_{q_i,D}$ most likely
- If query words are missing from document, score will be zero
  - Missing 1 out of 4 query words same as missing 3 out of 4

Smoothing

- Document texts are a *sample* from the language model
  - Missing words should not have zero probability of occurring
- *Smoothing* is a technique for estimating probabilities for missing (or unseen) words
  - lower (or *discount*) the probability estimates for words that are seen in the document text
  - assign that "left-over" probability to the estimates for the words that are not seen in the text

Estimating Probabilities

- Estimate for unseen words is $\alpha_D P(q_i|C)$
  - $P(q_i|C)$ is the probability for query word *i* in the *collection* language model for collection *C* (background probability)
  - α_D is a parameter
- Estimate for words that occur is

$$(1 - \alpha_D)\, P(q_i|D) + \alpha_D\, P(q_i|C)$$

- Different forms of estimation come from different α_D

Jelinek-Mercer Smoothing

- α_D is a constant, λ
- Gives estimate of

$$p(q_i|D) = (1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|}$$

- Ranking score

$$P(Q|D) = \prod_{i=1}^{n}((1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|})$$

- Use logs for convenience
  - accuracy problems multiplying small numbers

$$\log P(Q|D) = \sum_{i=1}^{n}\log((1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|})$$

Compare with *tf.idf*

$$\log P(Q|D) = \sum_{i=1}^{n} \log((1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|})$$

$$= \sum_{i:f_{q_i,D}>0} \log((1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|}) + \sum_{i:f_{q_i,D}=0} \log(\lambda\frac{c_{q_i}}{|C|})$$

$$= \sum_{i:f_{q_i,D}>0} \log \frac{((1-\lambda)\frac{f_{q_i,D}}{|D|} + \lambda\frac{c_{q_i}}{|C|})}{\lambda\frac{c_{q_i}}{|C|}} + \sum_{i=1}^{n} \log(\lambda\frac{c_{q_i}}{|C|})$$

$$\overset{rank}{=} \sum_{i:f_{q_i,D}>0} \log\left(\frac{((1-\lambda)\frac{f_{q_i,D}}{|D|}}{\lambda\frac{c_{q_i}}{|C|}} + 1\right)$$

- proportional to the term frequency, inversely proportional to the collection frequency

Dirichlet Smoothing

- α_D depends on document length

$$\alpha_D = \frac{\mu}{|D|+\mu}$$

- Gives probability estimation of

$$p(q_i|D) = \frac{f_{q_i,D}+\mu\frac{c_{q_i}}{|C|}}{|D|+\mu}$$

- and document score

$$\log P(Q|D) = \sum_{i=1}^{n} \log \frac{f_{q_i,D}+\mu\frac{c_{q_i}}{|C|}}{|D|+\mu}$$

Query Likelihood Example

- For the term “president”
  - $f_{qi,D} = 15$, $c_{qi} = 160{,}000$
- For the term “lincoln”
  - $f_{qi,D} = 25$, $c_{qi} = 2{,}400$
- document |d| is assumed to be 1,800 words long
- collection is assumed to be 10^9 words long
  - 500,000 documents times an average of 2,000 words
- $\mu = 2{,}000$

Query Likelihood Example

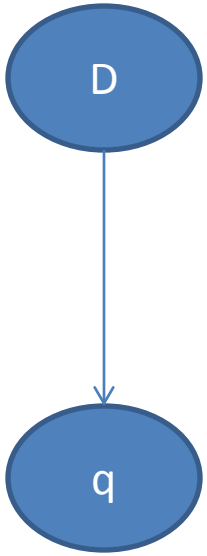
$$
\begin{aligned}
QL(Q,D) &= \log \frac{15 + 2000 \times (1.6 \times 10^5 / 10^9)}{1800 + 2000} \\
&\quad + \log \frac{25 + 2000 \times (2400/10^9)}{1800 + 2000} \\
&= \log(15.32/3800) + \log(25.005/3800) \\
&= -5.51 + -5.02 = -10.53
\end{aligned}
$$

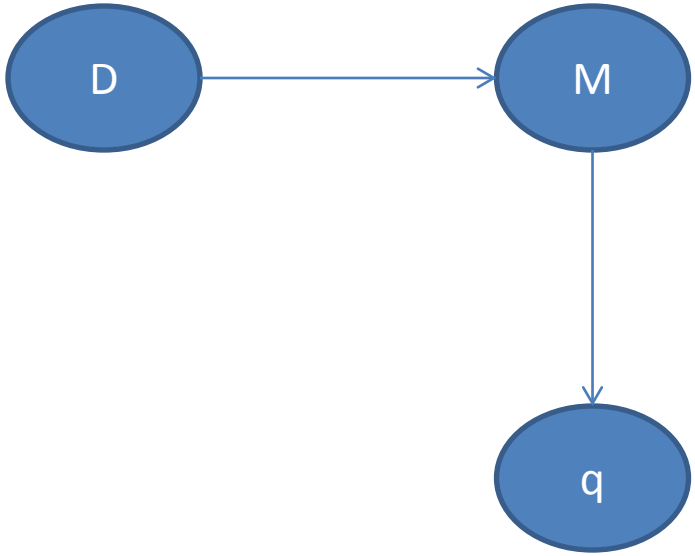
- Negative number because summing logs of small numbers

Query Likelihood Example

Frequency of "president"	Frequency of "lincoln"	QL score
15	25	-10.53
15	1	-13.75
15	0	-19.05
1	25	-12.99
0	25	-14.40

Going beyond tf.idf





$$p(q|D) = \frac{f_{q,D}}{|D|}$$

$$p(q|D) = \sum_{M} p(q|M)p(M|D)$$

$$p(q|M) = \frac{f_{q,M}}{|M|}$$

$$p(D|M) = \prod_{w}^{D} p(w|M)$$